Satoku Matrix Oreganography

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Wolfgang Scherer

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Abstract

Oreganography is the art of encrypting information in weedy ways for herbal recreation.

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1 Satoku Matrix

A satoku matrix is an inverted adjacency matrix preserving clause boundary information for implementing a requirement update algorithm (see figure 1.3).

A satoku matrix S is formally defined as a sequence of *cell-matrix rows* c_i , $0 \le i < |S|$, where a *cell-matrix row* c_i consists of *cells* c_{i_g} , $0 \le g < |S|$. A *cell* c_{i_g} consists of *cell rows* $r_{i_{j_g}}$, $0 \le j < |c_{i_g}|$, where a *cell row* $r_{i_{j_g}}$ consists of *states* $s_{i_{j_{g_h}}}$, $0 \le h < |r_{i_{j_g}}|$. A *state* $s_{i_{j_{g_h}}}$ represents a *conflict relationship* CFR with the *state* $s_{g_{h_{i_j}}}$, where a *conflict relationship* is either *possible* = 1 or *impossible* = 0. A *state row* s_{i_j} , $0 \le j < |c_{i_i}|$ is the sequence of *cell rows* $r_{i_{j_g}}$. The index scheme is summarized in table 1.1.

indexed state entity	description
C cell-matrix-row	row c_i of cells (cell-matrix-row)
C cell-matrix-row cell-matrix-column	single cell c_{i_g}
T cell-matrix-row cell-row cell-matrix-column	cell row $r_{i_{j_g}}$ containing all CFR states between an atomic state $s_{i_{j_{i_j}}}$ and an atomic cell c_{g_g}
S cell-matrix-row cell-row	state row s_{i_j} of all cell rows $r_{i_{j_g}}$ containing all singular states for an atomic state $s_{i_{j_{i_j}}}$
S cell-matrix-row cell-row cell-matrix-column cell-column	singular state $s_{i_{jg_h}}$

table 1.1: Satoku Matrix Index Scheme

For better readability, if a *cell row* $r_{i_{j_g}}$ contains more than one 1-state, all 1-states in $r_{i_{j_g}}$ are represented by a dash (-) (see figure 1.1).

Р					
s_{0_0}	$1 \circ \circ$				c_0
$s_{0_{1}}$	o 1 o	0			
s_{0_2}	$\circ \circ 1$		-0-	$0\ 1$	
s_{1_0}	-0 -	1000	0		c_{1_2}
$s_{1_{1}}$		0100	-0-	$0\ 1$	
s_{1_2}		0010		$1 \ 0$	
$s_{1_{3}}$		0001	0		
s_{20}		0	$1 \circ \circ$		s_{20}
s_{2_1}	0	-0	010		$s_{2_{1_0}}$
s_{2_2}		0	001		$s_{2_{2_{1_3}}}$
s_{30}	0	-0		1 0	
s_{3_1}		0-		o 1	$r_{3_{1_2}}$

figure 1.1: Satoku Matrix Index Scheme Example

Merging a sequence of *state rows* S into a *state row* s_{i_j} , denoted as $Mrg(s_{i_j}, S)$, is defined by the algorithm in figure 1.2. It returns the number of $1 \rightarrow 0$ transitions performed.

$$\begin{split} & \operatorname{def} \operatorname{Mrg}(s_{i_j}, S) \colon \\ & \operatorname{transitions} \leftarrow 0 \\ & \operatorname{for each state row } s_{g_h} \in S \colon \\ & \operatorname{for each state row } s_{g_h} \in S \colon \\ & \operatorname{for each state } s_{i_{j_{e_f}}}, \ 0 \leq e < |\mathbb{S}|, \ 0 \leq f < |r_{i_{j_e}}| \colon \\ & \operatorname{if } s_{i_{j_{e_f}}} \neq 0 \text{ and } s_{g_{h_{e_f}}} = 0 \colon \\ & s'_{i_{j_{e_f}}} = 0, \ \operatorname{transitions} += 1 \\ & \operatorname{if } s_{e_{f_{i_j}}} \neq 0 \colon \\ & s'_{e_{f_{i_j}}} = 0, \ \operatorname{transitions} += 1 \\ & \operatorname{return transitions} \end{split}$$

figure 1.2: Merge State Rows S Into State Row s_{i_i}

The requirement update algorithm in figure 1.3 distributes conflict relationships into all *cell rows* which have a single 1-state. After applying the requirement update algorithm, the satoku matrix S is called *consolidated*.

 $\begin{array}{l} \text{transitions} \leftarrow 1 \\ \text{while transitions} > 0: \\ \text{transitions} \leftarrow 0 \\ \text{for each state row } s_{i_j}: \\ \text{for each cell row } r_{i_{j_g}}, \ i \neq g: \\ \text{if there is only a single 1-state } s_{i_{j_{g_h}}} = 1, \ \sum_{f=0}^{|r_{i_{j_g}}|-1} s_{i_{j_{g_f}}} = 1: \\ \text{transitions} += \mathbf{Mrg}(s_{i_j}, s_{g_h}) // \text{merge state row } s_{g_h} \text{ into state row } s_{i_j} \end{array}$

figure 1.3: Requirement Update Algorithm

The consolidated version of the Satoku Matrix Index Scheme Example from figure 1.1 is shown in figure 1.4. Notably $s_{0_{2_{3_1}}}$ triggered a merge of s_{3_1} into s_{0_2} . This led to state $s_{3_{1_{2_2}}}$ causing $1 \rightarrow 0$ transitions of state $s_{0_{2_{1_2}}}$ and state $s_{1_{2_{0_2}}}$.

Р				
s_{0_0}	1 0 0			
s_{0_1}	010	0		
s_{0_2}	001	0-	-0-	01
s_{1_0}	-0-	1000	0	
$s_{1_{1}}$		0100	-0-	01
s_{1_2}	0	0010		10
s_{1_3}		0001	0	
s_{2_0}		0	1 0 0	
$s_{2_{1}}$	0	-0	010	
s_{2_2}		0	001	
s_{3_0}	0	-0		1 0
s_{3_1}		0-		o 1

figure 1.4: Consolidated Satoku Matrix Index Scheme Example

2 Mapping a CNF Formula to a Satoku Matrix

A CNF formula F is a conjunction of m disjunctive clauses C_i each containing k_i literals l_j , where a literal l_j is a negated or unnegated boolean variable:

$$F = \bigwedge_{i=0}^{m-1} C_i, \ m = |F|, \quad C_i = \bigvee_{j=0}^{k_i-1} l_j, \ k_i = |C_i|, \quad m, k_i \in \mathbb{N}_0.$$

Mapping a CNF formula F to a satoku matrix S with the algorithm in figure 2.1 results in an *unconsolidated* satoku matrix S.

$$\begin{split} \text{for each clause } C_i \text{ of CNF formula } F, \ 0 \leq i < |F|: \\ \text{extend each state row } s_{e_f}, \text{ by } |C_i| \ 1\text{-states, } 0 \leq e < i, \ 0 \leq f < |c_{e_e}| \\ \text{add } |C_i| \text{ state rows with } \sum_{n=0}^i |C_n| \ 1\text{-states each} \\ \textit{// process at-most-1 constraints} \\ \text{for each cell row } r_{i_{j_i}}, \ 0 \leq j < |C_i| - 1: \\ \text{for each state } s_{i_{j_{i_h}}}, \ j < h < |C_i|: \\ s_{i_{j_{i_h}}} \leftarrow 0 \\ s_{i_{h_{i_j}}} \leftarrow 0 \\ \textit{// process conflict relationships} \\ \text{for each state row } s_{i_j}, \ 0 \leq i < |F|, \ 0 \leq j < |C_i|: \\ \text{for each cell row } r_{i_{j_g}}, \ i < g < |F|: \\ \text{for each state } s_{i_{j_{g_h}}}, \ 0 \leq h < |C_g|: \\ \text{if } C_{i_j} \land C_{g_h} = 0: \\ s_{i_{j_{g_h}}} \leftarrow 0 \\ s_{g_{h_{i_j}}} \leftarrow 0 \\ \end{cases}$$

figure 2.1: Mapping a CNF Formula to a Satoku Matrix

3 Mapping a Satoku Matrix to a CNF Formula

See figure 3.1 for an algorithm to map a satoku matrix to a CNF formula.

start with empty CNF formula F (a conjunctive clause) for each cell c_{i_i} , $0 \le i < |\mathbb{S}|$: add an empty disjunctive clause C_i to Ffor each cell row $r_{i_{j_i}}$, $0 \le j < |c_{i_i}|$: add an unnegated variable v_{i_j} to clause C_i for each state row s_{i_j} , $0 \le i < |\mathbb{S}|$, $0 \le j < |c_{i_i}|$: with cell row $r_{i_{j_i}}$: for each state $s_{i_{j_{i_f}}}$, $j < f < |r_{i_{j_i}}|$: if $s_{i_{j_{i_f}}} = 0$: to express the conclusion $v_{i_j} \to \neg v_{i_f}$, add the disjunctive clause $\neg v_{i_j} \lor \neg v_{i_f}$ to |F|for each cell row $r_{i_{j_e}}$, $i < e < |\mathbb{S}|$: for each state $s_{i_{j_{e_f}}} = 0$: add the disjunctive clause $\neg v_{i_j} \lor \neg v_{e_f}$ to |F|if $s_{i_{j_{e_f}}} = 0$: add the disjunctive clause $\neg v_{i_j} \lor \neg v_{e_f}$ to |F|

figure 3.1: Mapping a Satoku Matrix to a CNF Formula

4 Initial Information Encoding

Arbitrary pixel blocks like "*HAPPYEASTER!* <*smile* >*egg* >*egg* " can be encoded in the upper right half of a satoku matrix in a simple manner by flipping 1-states to 0 (see figure 4.1).

		1				
Р						
s_{0_0}	100000	-00-	00	-000	-000	-00
s_{0_1}	010000	-0 0	-00-	-0 0	-0 0	0-0-0
s ₀₂	001000	-0000-	-00000-	-0000	-0000	0
s_{0_3}	000100	-0 0	-0 0	-0	-0	0
s_{0_4}	000010	-0 0	-0 0	-0	-0	0
$s_{0_{5}}$	000001					
s_{1_0}		100000	00	00	-00000-	100000
s_{1_1}	000001	010000	-0	-0 0	-0	0
s_{1_2}	0	001000	-00	-00000-	-00000-	0
s_{1_3}	0	000100	-0	-0 0	0-	0
s_{1_4}	$0 \ 0 \ 0 \ 0 \ 0 \ 1$	000010	00	-0 0	-00000-	0
$s_{1_{5}}$		000001				
s_{20}			100000	00	-000	0
s21	-00000-	-000	010000	-0	-0 0	0
s_{2_2}	0 - 0	0 - 0 - 0 - 0 - 0	001000	-00	-0000	0
s23	0 - 0	00-	000100	-0	-0 - 0	
s_{2_4}	$-0\ 0\ 0\ 0$ – –		000010	00	-0 0	0
s_{25}			000001			
s_{3_0}				100000		
s_{3_1}	$0\ 0\ 0\ 0\ 0\ 1$	-0000-	-0000	010000	-0 0	00
$s_{3_{2}}$	0 - 0	0-0	0 - 0 - 0 - 0 - 0	001000		-00-
$s_{3_{3}}$	0 - 0	0-0	00-	000100	0 0	-00-
s_{34}	-0	-0000-		000010	-00000 -	00
s_{3_5}				000001		
s_{4_0}				0	100000	
s_{4_1}	$0\ 0\ 0\ 0\ 0\ 1$	000-0-	$0\ 0\ 0\ 0\ 0\ 1$	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	010000	00
s_{4_2}	0 - 0	0 - 0 - 0 - 0	0-0	0-	001000	-0 0
$s_{4_{3}}$	0 - 0	0 - 0 - 0 - 0	$0 - 0 \ 0$	0 - 0	000100	-0 0
s_{44}	-0	0-000-	-0 0 -	-0 0 -	000010	00
s_{4_5}				0	000001	
s_{50}						100000
s_{5_1}	0	0		00	00	010000
s_{5_2}	-0	0		-0 0 -	-0 0	001000
$s_{5_{3}}$	$0\ 0\ 0$ –	000001	$0 \ 0 \ 0 \ -0 \ -$	-0 0 -	-0 0 -	000100
$s_{5_{4}}$	-0	0		00	00	000010
s_{55}	0	0				000001

figure 4.1: Text Encoded in a Satoku Matrix

The satoku matrix in figure 4.1 can then be mapped to CNF formula (see figure 6.1). Since this formula is huge, only a reduced version is shown here in figure 4.2.

 $\begin{pmatrix} v_{00} \lor v_{01} \lor v_{02} \lor v_{03} \lor v_{04} \lor v_{05} \end{pmatrix} \land \begin{pmatrix} v_{10} \lor v_{11} \lor v_{12} \lor v_{13} \lor v_{14} \lor v_{15} \end{pmatrix} \land \begin{pmatrix} \neg v_{00} \lor \neg v_{01} \end{pmatrix} \land \\ (\neg v_{00} \lor \neg v_{02}) \land (\neg v_{00} \lor \neg v_{03}) \land (\neg v_{00} \lor \neg v_{04}) \land (\neg v_{00} \lor \neg v_{05}) \land (\neg v_{00} \lor \neg v_{11}) \land (\neg v_{00} \lor \neg v_{14}) \land (\neg v_{01} \lor \neg v_{02}) \land (\neg v_{01} \lor \neg v_{03}) \land \\ (\neg v_{01} \lor \neg v_{04}) \land (\neg v_{01} \lor \neg v_{05}) \land (\neg v_{01} \lor \neg v_{11}) \land (\neg v_{01} \lor \neg v_{14}) \land (\neg v_{02} \lor \neg v_{03}) \land (\neg v_{02} \lor \neg v_{04}) \land (\neg v_{02} \lor \neg v_{05}) \land (\neg v_{02} \lor \neg v_{11}) \land \\ (\neg v_{02} \lor \neg v_{12}) \land (\neg v_{02} \lor \neg v_{13}) \land (\neg v_{02} \lor \neg v_{14}) \land (\neg v_{02} \lor \neg v_{03}) \land (\neg v_{02} \lor \neg v_{04}) \land (\neg v_{02} \lor \neg v_{05}) \land (\neg v_{02} \lor \neg v_{14}) \land \\ \end{pmatrix}$

figure 4.2: Reduced CNF Formula of Mapped Satoku Matrix

Mapping the CNF formula in figure 4.2 to an unconsolidated satoku matrix, effectively hides the encoded information (see figure 4.3).

$ \begin{array}{c} \mathbf{x}_{0_1} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0$	Р																 					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	s_{0_0}	100000		0-	0-	0 -	0-	0-	0-	0-							 					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				- 0								0-		0 -	0-	0-	 					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$																		0 -				0-
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$																						
								- 0						-0				-0				
$\begin{array}{c} s_{1_1} \\ s_{1_2} \\ s_{1_3} \\ s_{1_4} \\ s_{1_5} \\ s_{1_6} \\$			100000														 					
$\begin{array}{c c c c c c c c c c c c c c c c c c c $									-0						-0		 		-0			
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	s_{1_2}																			-0		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$																					-0	
$ \begin{array}{c} \hline \mathbf{x}_{20} \\ \mathbf{x}_{21} \\ \mathbf{x}_{31} \\ \mathbf{x}_{31$										-0						-0						-0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	015		000001												1							
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$																						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$																						
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		~																				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$																						
$\begin{array}{cccccccccccccccccccccccccccccccccccc$						-																
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							1.0	-														
$\begin{array}{c} \mathbf{x}_{6_{0}} & 0_{0} & \mathbf{x}_{0} & \mathbf{x}_{$																						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$								1.0									 					
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$																	 					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		0							10								 					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			-0														 					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	880	0								1 0							 					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			0-							$\circ 1$							 					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		-0									$1 \circ$						 					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		0									o 1						 					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	s_{10_0}											1 o					 					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	s_{10_1}	0										o 1					 					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	s_{11_0}																					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	s ₁₁₁																 					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$																						
$\begin{array}{cccccccccccccccccccccccccccccccccccc$																						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		-0															 					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$																1.	 					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			0_																			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$																						
$\begin{array}{cccccccccccccccccccccccccccccccccccc$																						
$\begin{array}{cccccccccccccccccccccccccccccccccccc$																	10					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$																						
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$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			-0														 		$\circ 1$			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	s190	0															 			1 0		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			0														 			o 1		
	s_{20_0}																 					
821 0 -	s_{20_1}		0														 				o 1	
	s_{21_0}	0															 					10
$\underline{s_{2I_1}} - \cdots - \underline{0} - 0$	s_{21_1}		0-														 					o 1

figure 4.3: Reduced CNF Formula Mapped to Unconsolidated Satoku Matrix

Performing the requirement update (figure 1.3) to the satoku matrix in figure 4.3 results in a consolidated satoku matrix, which shows the restored original information (see figure 4.4).

Р																						
s ₀₀	100000	-00-	01	01	01	01	01	01	01													
801	010000 001000	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	10	 10						$\begin{array}{c} 0 \ 1 \\ 1 \ 0 \end{array}$	01	01	01	01	01	 01	 0 1	${01}$	 01	01	01	${01}$
$s_{0_2} \\ s_{0_3}$	0001000				10						$10^{}$					101						
s ₀₄	000010					10						$1 \ 0$					$1 \ 0$					
s_{0_5}	000001						$1 \ 0$						$1 \ 0$					$1 \ 0$				
s_{1_0}		100000																				
811 81	000	010000 001000						10						10					10	10		
$s_{1_2} \\ s_{1_3}$		0001000																			10	
s_{1_4}	000	000010							$1 \ 0$						$1 \ 0$							$1 \ 0$
s_{15}		000001																				
s_{2_0}	0		10																			
s_{2_1}	-0		o 1																			
830	0			10																		
s ₃₁	0			∘1 	1 0																	
$s_{4_0} \\ s_{4_1}$	0				01																	
850	0					10																
85 ₁	0-					o 1																
s_{60}	0						1 0															
s_{6_1}	0						o 1															
s_{7_0}	0							10														
s ₇₁		-0						o 1														
$s_{8_0} \\ s_{8_1}$	0	0-							1 o o 1													
	-0									10												
s_{9_1}	0									01												
s_{100}	-0										$1 \circ$											
$s_{10_{1}}$	0										o 1											
s_{11_0}	-0											10										
s ₁₁₁	0-											o 1										
8120	-00												$1 \circ 1 \circ 1$									
\$121 \$10	-0													1 0								
$s_{13_0} \\ s_{13_1}$		-0												01								
s ₁₄₀	-0														1 0							
s_{14_1}		0-													$\circ 1$							
s_{15_0}																10						
s_{15_1}	0															o 1						
8160	0																1 o o 1					
\$161 \$																		 1 o				
$s_{17_0} \\ s_{17_1}$	0																	01				
s ₁₈₀	0																		10			
s ₁₈₁		-0																	o 1			
s_{19_0}	0																			1 0		
s_{19_1}		0																		o 1		
s_{20_0}	0																				10	
s ₂₀₁	0	0																			0 1	
s_{21_0} s_{21_1}	0	0-																				1 o 0 1
·211		= 0 =																				- I

figure 4.4: Consolidated Satoku Matrix for Reduced CNF Formula

5 Advanced Information Hiding

Removing the at-least-1 clauses from the reduced CNF formula in figure 4.3 results in a set of 2-literal clauses (see figure 5.1). For the full formula in figure 6.1 without at-least-1 clauses see figure 7.1.

 $\begin{array}{c} (\neg v_{00} \lor \neg v_{01}) \land (\neg v_{00} \lor \neg v_{02}) \land (\neg v_{00} \lor \neg v_{03}) \land (\neg v_{00} \lor \neg v_{04}) \land (\neg v_{00} \lor \neg v_{05}) \land (\neg v_{00} \lor \neg v_{11}) \land (\neg v_{00} \lor \neg v_{14}) \land (\neg v_{01} \lor \neg v_{02}) \land (\neg v_{01} \lor \neg v_{03}) \land (\neg v_{01} \lor \neg v_{04}) \land (\neg v_{01} \lor \neg v_{11}) \land (\neg v_{01} \lor \neg v_{14}) \land (\neg v_{02} \lor \neg v_{03}) \land (\neg v_{02} \lor \neg v_{04}) \land (\neg v_{02} \lor \neg v_{05}) \land (\neg v_{02} \lor \neg v_{11}) \land (\neg v_{02} \lor \neg v_{11}) \land (\neg v_{02} \lor \neg v_{11}) \land (\neg v_{02} \lor \neg v_{13}) \land (\neg v_{02} \lor \neg v_{14}) \land (\neg v_{02} \lor \neg v_{03}) \land (\neg v_{02} \lor \neg v_{04}) \land (\neg v_{02} \lor \neg v_{05}) \land (\neg v_{02} \lor \neg v_{11}) \land (\neg v_{02} \lor \neg v_{12}) \land (\neg v_{02} \lor \neg v_{13}) \land (\neg v_{02} \lor \neg v_{14})$

figure 5.1: Reduced CNF Formula Without At-Least-1 Clauses

Producing a consolidated satoku matrix from the formula in figure 5.1 lacks any information whatsoever (see figure 5.2).

Р																				
s_{0_0}	1 0																			
$s_{0_{1}}$	o 1																			
s ₁₀		10																		
s11		o 1																		
s_{2_0}			10																	
s_{2_1}			o 1																	
s_{3_0}				10																
s_{3_1}				$\circ 1$																
s_{4_0}					10															
s_{4_1}					o 1															
s_{5_0}						10														
s_{51}						o 1														
s_{6_0}							1 0													
$s_{6_{1}}$							$\circ 1$													
s_{7_0}								10												
s_{7_1}								01												
s_{8_0}									1 0											
$s_{8_{1}}$									$\circ 1$											
s_{9_0}										10										
$s_{9_{1}}$										o 1										
s ₁₀₀											10									
s_{10_1}											01									
s_{11_0}												1 0								
s_{11_1}												01								
s_{12_0}													1 0							
s_{12_1}													o 1							
s_{13_0}														1 0						
s_{13_1}														$\circ 1$						
s_{140}															10					
s_{14_1}															o 1					
s_{150}																1 0				
s_{15_1}																o 1				
s_{160}																	1 0			
$s_{16_{1}}$																	o 1			
s_{17_0}																		1 0		
s_{17_1}																		o 1		
s_{18_0}																			10	
s_{18_1}																			o 1	
s_{19_0}																				1 0
s_{19_1}																				o 1

figure 5.2: Consolidated Satoku Matrix for Reduced CNF Formula Without At-Least-1 Clauses

When the variables of the CNF formula in figure 5.1 are mapped via 2-literal clauses of the form $(p \lor \neg p)$, only the conflict relationships between the clauses and variables become visible (see figure 5.3)

Р																																
s00	1 0																				0 -											
s_{0_1}	o 1																					0-										
s_{1_0}		10																			0 -											
811 80		◦ 1 	1 0																		0 -		0-									
$s_{2_0} \\ s_{2_1}$			01																					0-								
s_{3_0}				10																	0 -											
s31 s40				∘ 1 	10																0 -				0-							
s_{4_1}					01																					0 -						
850						10															0 -											
851 860						◦ 1 	1 0														0 -							0-				
s_{6_1}							0 1																								0 -	
870								10														0-										
871 80								◦ 1 	 1 0													0-	0-									
$s_{8_0} \\ s_{8_1}$									01															0 -								
8 ₉₀										10												0-										
891 810										• 1 	 1 o														0-							
$s_{10_0} \\ s_{10_1}$											o 1															0 -						
s_{11_0}												10										0-										
8111 810												◦ 1 	 1 0															0-				
s_{12_0} s_{12_1}													o 1																		0 -	
s_{13_0}														10									0-									
8131 8140														◦1 	 1 o								0	0-								
s ₁₄															01										0 -							
s_{150}																10							0-									
$\frac{s_{15_1}}{s_{16_0}}$																∘ 1 	10						0-			0-						
s ₁₆₁																	0 1											0-				
8 ₁₇₀																		$\begin{array}{c} 1 \circ \\ \circ \end{array}$					0-						 0-			
$\frac{s_{17_1}}{s_{18_0}}$																			10				0-									
s ₁₈₁																			$\circ 1$											0 -		
8190																				1 o o 1			0-								 0-	
\$191	0-	0 -	0 -	0 -	0-	0-	0 -														1 0											
8200 8201																					01											
s_{21_0}	-0							0 -	0-	0 -	0 -	0 -	0-									10										
8 ₂₁₁								0						 0-				 0-		 0-		◦ 1 	 1 o									
s_{22_0} s_{22_1}																							01									
s_{230}			-0						- 0					- 0										10								
8231 8240				 -0						 -0					 -0									◦ 1 	 1 0							
8240 8241																									• 1							
825 ₀					- 0						-0					-0										$1 \circ 0$						
8251 8260																											10					
8261																											01					
827 ₀						-0						-0					-0											1 o o 1				
s ₂₇₁ s ₂₈₀																		0											10			
8280 8281																													01			
s ₂₉₀																			-0											10		
s ₂₉₁ s ₃₀₀							0						0							0										∘1 	 1 0	
s ₃₀₀ s ₃₀₁																															• 1	
8310																																10
s ₃₁₁																																o 1

figure 5.3: Unconsolidated Satoku Matrix of Reduced CNF With Mapped Variables

Only when the requirement update algorithm in figure 1.3 is applied to the unconsolidated satoku matrix in figure
5.3 the information reappears in the lower right mapped variable quadrant (see figure 5.4).

Р																																
	10																				0 1											
$s_{0_0} \\ s_{0_1}$	01																					0 1										
s_{1_0}		1 0																			0 1											
$\frac{s_{1_1}}{s_{2_0}}$		∘ 1 	 1 0																		01		01									
s_{2_1}			01																					0.1								
s_{3_0}				$1 \circ 1 \circ 1$																	0 1				${0}$							
831 840					1 0																0 1											
84 ₁					$\circ 1$																					$0 \ 1$						
$s_{5_0} \\ s_{5_1}$						$1 \circ 1 \circ 1$															01							${01}$				
- 51 860							1 0														0.1											
s_{6_1}							o 1																								0 1	
$s_{7_0} \\ s_{7_1}$								1 o o 1														01	01									
s_{8_0}									1 0													01										
$\frac{s_{8_1}}{s_{9_0}}$									◦ 1 	 1 0												01		01								
890 891										• 1															01							
s ₁₀₀											1 o o 1											0 1				 0 1						
$\frac{s_{10_1}}{s_{11_0}}$											○ I ——	 1 0										01										
s ₁₁₁												$\circ 1$																0.1				
s_{12_0} s_{12_1}													1 o o 1									01									01	
s ₁₃₀														1 0									0 1									
s_{13_1}														o 1										01								
s_{14_0} s_{14_1}															1 o o 1								01		$\frac{-}{01}$							
s_{150}																1 0							0.1									
8151																∘ 1 	 1 0						01			01						
$s_{16_0} \\ s_{16_1}$																	01											$0\ 1$				
8 ₁₇₀																		$1 \circ 1 \circ 1$					0 1						 0 1			
8171 8180																			1 0				01									
s_{18_1}																			o 1											01		
$s_{19_0} \\ s_{19_1}$																				1 o o 1			01								01	
s200	0.1	0.1	0.1	0.1	0.1	0.1	0 1														1 0	0.1	01	0.1	0.1	0.1		01			01	
$s_{20:}$																					01											
$s_{21_0} \\ s_{21_1}$	10							$ \begin{array}{c} 0 \\ \end{array} $	01	01	01	$ \begin{array}{c} 0 \\ \end{array} $	01								01	1 o 0 1	01	01	01	01		01			01	
s_{220}		1.0						$1 \ 0$						01	01	01	0 1	0 1	01	01	0.1	0.1	10	0.1	0.1	0.1		01	0.1	0.1	0.1	
$\frac{s_{22_1}}{s_{23_0}}$			10						10					 10							01	0.1	◦ 1 0 1	 1 0								
8230 8231																								o 1								
$s_{24_0} \\ s_{24_1}$				10						10					10						0 1	0 1	0 1		$1 \circ 1 \circ 1$							
8241 8250					10						10					10					01	01	01			1 0						
s_{25_1}																										o 1						
$s_{26_0} \\ s_{26_1}$																											1 o o 1					
827 ₀						10						10					10				01	01	01					1 0				
s_{27_1}																		 10										01	 1 0			
$s_{28_0} \\ s_{28_1}$																							01						1 o o 1			
s_{290}																			$1 \ 0$				0.1							10		
8291 8300							 1 0						 10							 10	01	01								◦ 1 	 1 0	
s ₃₀₀ s ₃₀₁																															01	
8310																																1 o o 1
8311																																- 1

figure 5.4: Consolidated Satoku Matrix of Reduced CNF With Mapped Variables

The mapped variable quadrant of the satoku matrix derived from the full CNF formula in figure 7.1 already hints strongly at the encoded message (see figure 5.5).

Р																																				
800	10	0.1	01	01	0.1	01		0.1			01				01	01				0 1	0 1	0 1				0.1	01	0.1				01				01
801 810	◦ 1 0 1	1 o o 1	0 1	0 1	0 1	01		01			0 1			01			01			0 1			0 1			01			0 1				0 1		01	
811 820	01	01	1 0	0 1	01	01		01	0 1	01	01			01	0 1	0 1	01			01	0 1	0 1				01	01	0 1						0 1		
821 830	01	01	◦ 1 0 1	 1 0	01	01		01			01			01			01			01						01								01		
831 840		01	01	◦ 1 0 1	 1 0	01		 01			01			01			01			 0 1														01		
841 850		 01	 01	 01	◦ 1 0 1	 1 o																														
$s_{5_{1}}$						o 1	 1 0	 0 1		 01	 01	 01			 0 1	 01					 0 1	 01					 01	 0 1	 01			 0 1	 0 1	 0 1	 01	
860 861							o 1																													
$s_{7_0} \\ s_{7_1}$	01	$ \begin{array}{c} 0 \\ \end{array} $	$ \begin{array}{c} 0 \\ \end{array} $	$ \begin{array}{c} 0 \\ \end{array} $	01		01	1 o o 1	01	$ \begin{array}{c} 0 \\ \end{array} $	01	01		01						01			01			01								01		
$s_{8_0} \\ s_{8_1}$			01				01	01	1 o o 1	01 	01	01		01	01					01	01	01	01			01	01	01	01					01		
890 891			01				01	01	01	1 o o 1	01	01		01						01			01						01					01		
$s_{10_0} \\ s_{10_1}$	01	01	01	01	01		01	01	01	01	1 o o 1	01			01	01				01			01			01	01	01	01					01		
s_{110}							01	01	01	01	01	1 o o 1																								
8111 8120													1 0	0.1	0.1	01	01	01			0.1	0 1				0.1	01	0.1						0 1		
s ₁₂₁ s ₁₃₀		 0 1	01	 0 1	01			01	01	01			01 01	 1 0	01	 0 1	01	01		 0 1						01			01					01		
s ₁₃₁			 01				01				 01		01	• 1 0 1	 1 o	 01	 0 1	 01		 01	 0 1						 01	01						 01		
8141 8150	 01		 01				01				 01		01	 01	◦ 1 0 1	 1 0	 01	 01		 01								01								
s_{15_1}		 01	 01	 0.1											 0 1	0 1 0 1		 01								 0 1			 01					 0.1		
s_{16_0} s_{16_1}																	$\circ 1$																			
$s_{170} \\ s_{171}$													01	01	01	01	01	1 o o 1																		
$s_{18_0} \\ s_{18_1}$																			$1 \circ 1 \circ 1$	01 	01	01	01	01												
$s_{19_0} \\ s_{19_1}$	01	$^{0\ 1}_{}$	$^{0\ 1}_{}$	$^{0\ 1}_{}$	01			01	01	$^{0\ 1}_{}$	$^{0\ 1}_{}$			01	01	$^{0\ 1}_{}$			01	$\begin{array}{c} 1 \circ \\ \circ \end{array}$	01 	01	01	01		01			01				01	$^{0\ 1}_{}$		
8200 8201	01		$^{0\ 1}_{}$				01		01				01		01		01		01	01	$1 \circ 0 \circ 1$	01	01	01								01			01	
s ₂₁₀ s ₂₁₁	01		01				01		01				01				01		01	01	01	1 o o 1	01	01	01					01		01			01	
8220 8221		01						01	01	01	01								01	01	01	01	10	01		01	01	01	01				01	01		
s ₂₃₀																			0.1	0 1	0 1	0 1	0 1	1 o o 1												
8231 8240																						0 1			1 0	0 1	01	0 1	0 1	01						
8241 8250	01	01	01	 0 1	01		01	01	01		01		01	01	01	 0 1	01			 0 1			01		◦ 1 0 1	 1 0	01	01	01	01			01	01		
8251 8260			 01								 01				 01											◦ 1 0 1	 1 0	01		 01		 01			 01	
8261 8270			 01				01				 01		 01		 01	 01									 01		∘1 01	 1 o							 0.1	
s_{27_1}																												0.1					 0 1	 0 1		
$s_{28_0} \\ s_{28_1}$		01					01		01	01	01			01			01			01			01		01	01	01	01	10	01						
$s_{29_0} \\ s_{29_1}$																						01			01	01	01	01	01	1 o o 1						
$s_{30_0} \\ s_{30_1}$																															$\begin{array}{c} 1 \circ \\ \circ 1 \end{array}$	01	$ \begin{array}{c} 0 \\ \end{array} $	01 	01	01
8310 8311	01						01														$^{0\ 1}_{}$	$ \begin{array}{c} 0 \\ \end{array} $					$^{01}_{}$	$ \begin{array}{c} 0 \\ \end{array} $			01	$\begin{array}{c} 1 \circ \\ \circ \end{array}$	$ \begin{array}{c} 0 \\ \end{array} $	$^{0\ 1}_{}$	01	01
8320 8321		01					01																01											01		
s_{330}			01	01			01	01		01	0.1				01		$0 \ 1$			0.1			01						01		0.1	0.1	0.1		0.1	01
8331 8340		0.1					0.1														0.1						0.1	01					0.1	0.1	10	0.1
8341 8350	0.1						01																								0.1	0.1	0.1	 0 1	0.1	1 0
8 ₃₅₁																																				o 1

figure 5.5: Mapped Variable Quadrant of Full CNF Formula Without At-Least-1 Clauses

The original message can be restored by adding the at-least-1 clauses manually to the mapped variable quadrant from figure 5.4 and making exactly one mapped variable required in the lower right quadrant for each literal of the at-least-1 clauses (see figure 5.6).

						1															
Р																					
s_{0_0}	1 0	$0\ 1$	$0\ 1$	$0\ 1$	$0 \ 1$	01		$0\ 1$			$0 \ 1$				$0 \ 1$	$0 \ 1$					
s ₀₁	o 1																		0		
s_{1_0}	01	10	$0\ 1$	01	01	01		$0\ 1$			01			01			01				
s ₁₁		o 1																	-0		
82 ₀	01	01	10	01	01	01		01	01	01	01			01	01	01	01				
\$2 ₁			01																		
830 80	01	01	01	1 o o 1															0		
831	01	01	01		1 0														0		
$s_{4_0} \\ s_{4_1}$					01														0_		
s ₅₀	01	01	01			10															
s_{51}						01													0		
s ₆₀							10														
s_{6_1}							$\circ 1$													0	
s70	$0 \ 1$	$0\ 1$	$0 \ 1$					1 0													
s ₇₁								o 1												-0	
s_{8_0}			$0\ 1$						1 0												
$s_{8_{1}}$									o 1											0	
s_{9_0}			$0\ 1$							10											
s_{9_1}										01										0	
s_{10_0}	0 1	01	01								10										
s ₁₀₁											01									0-	
8 ₁₁₀												1 o o 1								0	
s ₁₁₁																				0	
s_{12_0} s_{12_1}													1 o o 1								0
		01	01											10							·
$s_{13_0} \\ s_{13_1}$														01							-0
s ₁₄₀	01		01												10						
S ₁₄₀															01						0
s_{150}	0.1		$0\ 1$													1 0					
s_{15_1}																$\circ 1$					0
s_{16_0}		$0 \ 1$	$0\ 1$														1 0				
s_{16_1}																	o 1				0-
s_{170}																		10			
s_{17_1}																		o 1			0
s_{18_0}	-0																		100000		
s_{18_1}		-0																	010000		
818 ₂			-0	0															001000		
$s_{18_3} \\ s_{18_4}$					-0														0000100		
s ₁₈₅						-0													000001		
s_{190}							-0													100000	
s ₁₉₁								-0												010000	
s_{19_2}									- 0											001000	
8193 810										-0	0									000100 000010	
$s_{19_4} \\ s_{19_5}$												$-0^{$								0000010	
s ₂₀₀													- 0								100000
s_{20_1}														-0							010000
s202															-0						001000
s_{20_3}																-0					000100
8204																	-0	0			000010
$s_{20_{5}}$																		- 0			000001

figure 5.6: At-Least-1 Clauses Manually Added to Mapped Variable Quadrant

Р																					
s_{0_0}	1 0	$0 \ 1$	$0 \ 1$	$0 \ 1$	$0 \ 1$	0.1		$0 \ 1$			$0 \ 1$				$0 \ 1$	$0 \ 1$			$1 \ 0 \ 0 \ 0 \ 0 \ 0$	-0 0	00
$s_{0_{1}}$	$\circ 1$																		0		
s_{1_0}	$0 \ 1$	$1 \circ$	$0 \ 1$	$0 \ 1$	$0 \ 1$	0.1		$0 \ 1$			$0 \ 1$			$0 \ 1$			01		$0\ 1\ 0\ 0\ 0\ 0$	-0 0	-0 0 -
$s_{1_{1}}$		o 1																	- 0		
s_{2_0}	$0 \ 1$	$0 \ 1$	1 0	$0 \ 1$	$0 \ 1$	01		$0 \ 1$	$0 \ 1$	$0 \ 1$	$0 \ 1$			$0 \ 1$	$0 \ 1$	01	01		001000	-0000-	-0000-
s_{2_1}			o 1																0		
s_{30}	$0 \ 1$	0.1	01	1 o															000		
s_{3_1}				$\circ 1$															0		
s_{40}	01	$0 \ 1$	01		10														000		
s41					o 1														0-		
\$ ₅₀	01	01	01			10													000		
s_{5_1}						o 1													0		
\$60							10														
s ₆₁							01													0	
870	01	01	01					10											000		
s ₇₁								01												-0	
s80			01						10										0		
880 881									01											0	
			01							10									0		
$s_{9_0} \\ s_{9_1}$										01										0	
	0.1	01	01								1 0								000	•	
$\frac{s_{10_0}}{s_{10_1}}$											01									0_	
												10									
S110												01								0	
S ₁₁₁													1							0	
8120													1 o o 1								0
s ₁₂₁														1					0.0		0
8130		01	01											1 o o 1					-00		-0
s ₁₃₁	0.1																		0 0		_ 0
S140	01		01												1 o 0 1				0-0		
8141	0.1																		0 0		0
s ₁₅₀	01		01													1 o o 1			0-0		0
S 151																			0.0		0
s ₁₆₀		01	01														1 o 0 1		-00		0_
s ₁₆₁																					0-
8 ₁₇₀																		1 o o 1			0
s ₁₇₁																		01			0
s_{180}	10	01	01	01	01	01		01			01				$0\ 1$	$0\ 1$			100000	-00-	00
8181	01	10	01	01	01	01		01			01			01			$ \begin{array}{c} 0 \\ 0 \\ 1 \end{array} $		010000	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	-0 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 -
8182 8	01	$ \begin{array}{c} 0 \\ 0 \\ 1 \end{array} $	$ \begin{array}{c} 1 & 0 \\ 0 & 1 \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} $	01	01		01	01	01	01			01	01	$ \begin{array}{c} 0 \\ \end{array} $	01		001000	-0000-	-0000-
8183	$ \begin{array}{c} 0 \\ 0 \\ 1 \end{array} $	$01 \\ 01$	$01 \\ 01$		10														0000100		
$s_{18_4} \\ s_{18_5}$		01	01			10													0000001		
							10													100000	
$s_{19_0} \\ s_{19_1}$	0.1	01	0.1					10											000	010000	
$s_{19_1} \\ s_{19_2}$			01						$1 \ 0$										0	001000	
S193			01							$1 \ 0$									0	000100	
s_{19_4}	$0\ 1$	$0\ 1$	$0\ 1$								$1\ 0$								000	000010	
s_{195}												$1 \ 0$								000001	
s_{20_0}													$1 \ 0$								100000
s_{20_1}		-												$1 \ 0$					-0.0		010000
s_{20_2}	01		01												$1 \ 0$				0-0		001000
8203	01		01													10	1.0		0-0		000100
8204		01	01														10	1.0	-00		000010
$s_{20_{5}}$																		10			000001

Running the requirement update algorithm from figure 1.3 consolidates the satoku matrix from figure 5.6 and restores the original message in the lower right Quadrant (see figure 5.7).

figure 5.7: Requirement Update Restores Original Message

Removing the mapped variables finally leaves an exact copy of the original message (see figure 5.8).
--

Р						
s_{0_0}	100000	-00-	00	-000	-000	-0 0
$s_{0_{1}}$	010000	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	0-0-
s_{0_2}	001000	-0000-	-0000-	-000	-0000	0
s_{0_3}	000100 000010	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	-0	-0	0
s_{04}	0000010	-00-	-00-	-0	-0	0
s ₀₅	000001					
$s_{1_{0}}$		100000	00	00	-00000-	$1\ 0\ 0\ 0\ 0\ 0$
$s_{1_{1}}$	000001	010000	-0	-00-	-0	0
$s_{1_{2}}$	0	001000	-00	-00000-	-00000-	0
$s_{1_{3}}$	0	000100	-0	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	0-	0
s_{1_4}	000001	000010	00	-00-	-00000-	0
s ₁₅		000001				
s_{20}			100000	00	-0000	0
s_{2_1}	-0000-	-000	010000	-0	-00-	0
s_{2_2}	0-0	0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 -	001000	-00	-0000	0
$s_{2_{3}}$	0-0	00-	000100	-0	-0-0	
s_{2_4}	-00000 -		000010	00	-00-	0
s_{25}			000001			
s_{3_0}				100000		
s_{3_1}	000001	-0000-	-000	010000	-0 0	00
$s_{3_{2}}$	0-0	0-0	0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 -	001000		-00-
s_{3_3}	0-0	0-0	00-	000100	0 0	-00-
s_{34}	-0	-0000-		000010	-00000-	00
$s_{3_{5}}$				000001		
s_{4_0}				0	100000	
s_{4_1}	000001	000-0-	000001	-0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0	010000	00
s_{4_2}	0-0	0 - 0 - 0 - 0 - 0	0-0	0-	001000	-00-
s_{4_3}	0-0	0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 -	0-00	0-	000100	-00-
s_{44}	-0	0-000-	-00-	-00-	000010	00
$s_{4_{5}}$				0	000001	
s_{50}						1 • • • • • •
s_{5_1}	0	0		00	00	010000
s_{52}	-0	0		-00-	-00-	001000
s_{53}		000001	$0 \ 0 \ 0 \ -0 \ -$	-00-	-00-	000100
s_{5_4}	-0	0		00	00	000010
s_{55}	0	0				000001

figure 5.8: Removing Mapped Variables Leaves Exact Copy of Original Message

6 Satoku Matrix Mapped to CNF Formula

 $(v_{00} \lor v_{01} \lor v_{02} \lor v_{03} \lor v_{04} \lor v_{05}) \land (v_{10} \lor v_{11} \lor v_{12} \lor v_{13} \lor v_{14} \lor v_{15}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{10} \lor v_{11} \lor v_{12} \lor v_{13} \lor v_{14} \lor v_{15}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25}) \land (v_{20} \lor v_{21} \lor v_{22} \lor v_{23} \lor v_{24} \lor v_{25} \lor v_$ $v_{30} \lor v_{31} \lor v_{32} \lor v_{33} \lor v_{34} \lor v_{35}) \land (v_{40} \lor v_{41} \lor v_{42} \lor v_{43} \lor v_{44} \lor v_{45}) \land (v_{50} \lor v_{51} \lor v_{52} \lor v_{53} \lor v_{54} \lor v_{55}) \land (v_{50} \lor v_{51} \lor v_{51} \lor v_{52} \lor v_{53} \lor v_{54} \lor v_{55}) \land (v_{50} \lor v_{51} \lor v_{51} \lor v_{52} \lor v_{53} \lor v_{54} \lor v_{55}) \land (v_{50} \lor v_{51} \lor v_{51} \lor v_{52} \lor v_{53} \lor v_{54} \lor v_{55}) \land (v_{50} \lor v_{51} \lor v_{51} \lor v_{52} \lor v_{53} \lor v_{54} \lor v_{55}) \land (v_{50} \lor v_{51} \lor v_{51} \lor v_{52} \lor v_{53} \lor v_{54} \lor v_{55}) \land (v_{50} \lor v_{51} \lor v_{51} \lor v_{52} \lor v_{53} \lor v_{54} \lor v_{55}) \land (v_{50} \lor v_{51} \lor v_{51} \lor v_{52} \lor v_{53} \lor v_{54} \lor v_{55}) \land (v_{50} \lor v_{51} \lor v_{51} \lor v_{52} \lor v_{53} \lor v_{54} \lor v_{55}) \land (v_{50} \lor v_{51} \lor v_{52} \lor v_{55} \lor v_{5} \lor v_{5$ $(\neg v_{00} \lor \neg v_{01}) \land (\neg v_{00} \lor \neg v_{02}) \land (\neg v_{00} \lor \neg v_{03}) \land (\neg v_{00} \lor \neg v_{04}) \land (\neg v_{00} \lor \neg v_{05}) \land (\neg v_{00} \lor \neg v_{11}) \land (\neg v_{00} \lor \neg v_{14}) \land (\neg v_{00} \lor \neg v_{22}) \land (\neg v_{00} \lor \neg v_{14}) \land (\neg v_{14} \lor (\neg v_{14} \lor \neg v_{14}) \land (\neg v_{14} \lor (\neg v_{14} \lor (\neg v_{14} \lor \neg v_{14}) \land (\neg v_{14} \lor (\neg v$ $(\neg v_{00} \lor \neg v_{23}) \land (\neg v_{00} \lor \neg v_{31}) \land (\neg v_{00} \lor \neg v_{32}) \land (\neg v_{00} \lor \neg v_{33}) \land (\neg v_{00} \lor \neg v_{41}) \land (\neg v_{00} \lor \neg v_{42}) \land (\neg v_{00} \lor \neg v_{43}) \land (\neg v_{00} \lor \neg v_{51}) \land (\neg v$ $\begin{array}{c} (-v_{00} \lor v_{23}) \land (-v_{00} \lor v_{31}) \land (-v_{01} \lor v_{03}) \land (-v_{01} \lor v_{04}) \land (-v_{01} \lor v_{03}) \land (-v_{01} \lor v_{04}) \land (-v_{01} \lor v_{03}) \land (-v_{01} \lor v_{01}) \land ($ $(\neg v_{02} \lor \neg v_{04}) \land (\neg v_{02} \lor \neg v_{05}) \land (\neg v_{02} \lor \neg v_{11}) \land (\neg v_{02} \lor \neg v_{12}) \land (\neg v_{02} \lor \neg v_{13}) \land (\neg v_{02} \lor \neg v_{14}) \land (\neg v_{02} \lor \neg v_{21}) \land (\neg v_{02} \lor \neg v_{22}) \land (\neg v_{02} \lor \neg v_{12}) \land (\neg v$ $\neg v_{02} \lor \neg v_{23}) \land (\neg v_{02} \lor \neg v_{24}) \land (\neg v_{02} \lor \neg v_{31}) \land (\neg v_{02} \lor \neg v_{32}) \land (\neg v_{02} \lor \neg v_{33}) \land (\neg v_{02} \lor \neg v_{41}) \land (\neg v_{02} \lor \neg v_{42}) \land (\neg v_{02} \lor \neg v_{43}) \land (\neg v_{02} \lor \neg v_{41}) \land (\neg v_{02} \lor \neg v_{42}) \land (\neg v_{02} \lor \neg v_{43}) \land (\neg v_{02} \lor \neg v_{41}) \land (\neg v_{02} \lor \neg v_{42}) \land (\neg v_{02} \lor \neg v_{43}) \land (\neg v_{02} \lor \neg v_{41}) \land (\neg v_{02} \lor \neg v_{42}) \land 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\land (\neg v_{14} \lor \neg v_{34}) \land (\neg v_{14} \lor v_{24}) \land (\neg v_{14$ $\neg v_{14} \lor \neg v_{42}) \land (\neg v_{14} \lor \neg v_{43}) \land (\neg v_{14} \lor \neg v_{44}) \land (\neg v_{14} \lor \neg v_{53}) \land (\neg v_{20} \lor \neg v_{21}) \land (\neg v_{20} \lor \neg v_{22}) \land (\neg v_{20} \lor \neg v_{23}) \land (\neg v_{20} \lor \neg v_{24}) \land (\neg v_{20} \lor \neg v_{23}) \land (\neg v_{20} \lor (\neg v_{23}) \land (\neg v_{23} \lor (\neg v_{23}) \land$ $(\neg v_{20} \lor \neg v_{25}) \land (\neg v_{20} \lor \neg v_{32}) \land (\neg v_{20} \lor \neg v_{33}) \land (\neg v_{20} \lor \neg v_{41}) \land (\neg v_{20} \lor \neg v_{42}) \land (\neg v_{20} \lor \neg v_{43}) \land (\neg v_{20} \lor \neg v_{53}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{22} \lor \neg v_{23}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{22} \lor \neg v_{23}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{22} \lor \neg v_{23}) \land (\neg v_{23} \lor 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\vee v_{53}) \land (\neg v_$ $\neg v_{23} \lor \neg v_{24}) \land (\neg v_{23} \lor \neg v_{25}) \land (\neg v_{23} \lor \neg v_{31}) \land (\neg v_{23} \lor \neg v_{41}) \land (\neg v_{23} \lor \neg v_{43}) \land (\neg v_{24} \lor \neg v_{25}) \land (\neg v_{24} \lor \neg v_{32}) \land (\neg v_{24} \lor \neg v_{33}) \land (\neg v_{24} \lor \neg v_{32}) \land (\neg v_{34} \lor \neg v_{34}) \land (\neg v_{34} \lor (\neg v_{34} \lor \neg v_{34}) \land (\neg v_{34} \lor (\neg v_{34} \lor \neg v_{34}) \land (\neg v_{34} \lor (\neg v_{34} \lor (\neg v_{34} \lor (\neg v_{34} \lor (\neg v_$ $\neg v_{24} \lor \neg v_{41}) \land (\neg v_{24} \lor \neg v_{44}) \land (\neg v_{24} \lor \neg v_{53}) \land (\neg v_{30} \lor \neg v_{31}) \land (\neg v_{30} \lor \neg v_{32}) \land (\neg v_{30} \lor \neg v_{33}) \land (\neg v_{30} \lor \neg v_{34}) \land (\neg v_{30} \lor \neg v_{35}) \land (\neg v_{30} \lor \neg v_{31}) \land (\neg v_{30} \lor \neg v_{31}) \land (\neg v_{30} \lor \neg v_{32}) \land 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v_{33} \land (\neg v_{32} \lor \neg v_{34}) \land (\neg v_{32} \lor \neg v_{35}) \land (\neg v_{32} \lor \neg v_{51}) \land (\neg v_{32} \lor \neg v_{51}) \land (\neg v_{33} \lor \neg v_{34}) \land (\neg v_{33} \lor \neg v_{35}) \land (\neg v_{33} \lor \neg v_{34}) \land (\neg v_{33} \lor \neg v_{35}) \land (\neg v_{33} \lor v_{35}) \land (\neg v_{33$ $\neg v_{33} \lor \neg v_{45}) \land (\neg v_{33} \lor \neg v_{51}) \land (\neg v_{33} \lor \neg v_{54}) \land (\neg v_{34} \lor \neg v_{35}) \land (\neg v_{34} \lor \neg v_{41}) \land (\neg v_{34} \lor \neg v_{42}) \land (\neg v_{34} \lor \neg v_{43}) \land (\neg v_{34} \lor \neg v_{44}) \land (\neg v_{34} \lor \neg v_{45}) \land (\neg v_$ $(\neg v_{34} \lor \neg v_{52}) \land (\neg v_{34} \lor \neg v_{53}) \land (\neg v_{40} \lor \neg v_{41}) \land (\neg v_{40} \lor \neg v_{42}) \land (\neg v_{40} \lor \neg v_{43}) \land (\neg v_{40} \lor \neg v_{44}) \land (\neg v_{40} \lor \neg v_{45}) \land (\neg v_{41} \lor \neg v_{42}) \land (\neg v_{40} \lor \neg v_{45}) \land (\neg v_{45} \lor (\neg v_{45}) \land (\neg v_{45} \lor (\neg v_{45}) \land (\neg$ $\neg v_{41} \lor \neg v_{43}) \land (\neg v_{41} \lor \neg v_{44}) \land (\neg v_{41} \lor \neg v_{45}) \land (\neg v_{41} \lor \neg v_{52}) \land (\neg v_{41} \lor \neg v_{53}) \land (\neg v_{42} \lor \neg v_{43}) \land (\neg v_{42} \lor \neg v_{44}) \land (\neg v_{42} \lor \neg v_{45}) \land (\neg v_{42} \lor \neg v_{43}) \land (\neg v_{43} \lor (\neg v_{43} \lor \neg v_{43}) \land (\neg v_{43} \lor (\neg v_{43} \lor (\neg v_{43} \lor \neg v_{43}) \land (\neg v_{43} \lor (\neg v_$ $\neg v_{42} \lor \neg v_{51}) \land (\neg v_{42} \lor \neg v_{54}) \land (\neg v_{43} \lor \neg v_{44}) \land (\neg v_{43} \lor \neg v_{55}) \land (\neg v_{43} \lor \neg v_{51}) \land (\neg v_{43} \lor \neg v_{54}) \land (\neg v_{44} \lor \neg v_{55}) \land (\neg v_{44} \lor \neg v_{55}) \land (\neg v_{43} \lor \neg v_{51}) \land (\neg v_{51} \lor (\neg v_{51}) \land (\neg v_{51} \lor (\neg v_{51}) \land (\neg v_{51}) \land (\neg v_{51} \lor (\neg v_{51}) \land (\neg v_{51} \lor (\neg v_{51}) \land (\neg v_{51} \lor (\neg v_{51}) \land (\neg v_{51} \lor$ $\neg v_{44} \lor \neg v_{53}) \land (\neg v_{50} \lor \neg v_{51}) \land (\neg v_{50} \lor \neg v_{52}) \land (\neg v_{50} \lor \neg v_{53}) \land (\neg v_{50} \lor \neg v_{54}) \land (\neg v_{50} \lor \neg v_{55}) \land (\neg v_{51} \lor \neg v_{52}) \land (\neg v_{51} \lor \neg v_{53}) \land (\neg v_{51} \lor (\neg v_{51} \lor \neg v_{53}) \land (\neg v_{53} \lor (\neg v_$ $(\neg v_{51} \lor \neg v_{54}) \land (\neg v_{51} \lor \neg v_{55}) \land (\neg v_{52} \lor \neg v_{53}) \land (\neg v_{52} \lor \neg v_{54}) \land (\neg v_{52} \lor \neg v_{55}) \land (\neg v_{53} \lor \neg v_{54}) \land (\neg v_{53} \lor \neg v_{55}) \land (\neg v_{54} \lor v_{55}) \land (\neg v_{54} \lor v_{55}) \land (\neg v_{54} \lor v_{55}) \land (\neg v_{55} \lor v_{55})$

figure 6.1: Satoku Matrix Mapped to CNF Formula

7 CNF Formula Without At-Least-1 Clauses

$(\neg v_{00} \lor \neg v_{01}) \land (\neg v_{00} \lor \neg v_{02}) \land (\neg v_{00} \lor \neg v_{03}) \land (\neg v_{00} \lor \neg v_{04}) \land (\neg v_{00} \lor \neg v_{05}) \land (\neg v_{00} \lor \neg v_{11}) \land (\neg v_{00} \lor \neg v_{14}) \land (\neg v_{00} \lor \neg v_{22}) \land (\neg v_{00} \lor \neg v_{10}) \land (\neg v_{10} \lor (\neg v_{10} \lor \neg v_{10}) \land (\neg v_{10} \lor (\neg v_{10} \lor (\neg v_{10} \lor \neg v_{10}) \land (\neg v_{10} \lor (\neg v$
$(\neg v_{00} \lor \neg v_{23}) \land (\neg v_{00} \lor \neg v_{31}) \land (\neg v_{00} \lor \neg v_{32}) \land (\neg v_{00} \lor \neg v_{33}) \land (\neg v_{00} \lor \neg v_{41}) \land (\neg v_{00} \lor \neg v_{42}) \land (\neg v_{00} \lor \neg v_{43}) \land (\neg v_{00} \lor \neg v_{51}) \land (\neg v_{00} \lor \vee v_{51}) \land (\neg v_{00} \lor \vee v_{51}) \land (\neg v_{00} \lor \vee v_{51}) \land (\neg v$
$(\neg v_{00} \lor \neg v_{55}) \land (\neg v_{01} \lor \neg v_{02}) \land (\neg v_{01} \lor \neg v_{03}) \land (\neg v_{01} \lor \neg v_{04}) \land (\neg v_{01} \lor \neg v_{05}) \land (\neg v_{01} \lor \neg v_{11}) \land (\neg v_{01} \lor \neg v_{14}) \land (\neg v_{01} \lor \neg v_{21}) \land (\neg v_{01} \lor \neg v_{12}) \land (\neg v_{01} \lor \vee v_{12}) \land (\neg v_{12} \lor \vee v_{12}) \land (\neg v$
$(\neg v_{01} \lor \neg v_{24}) \land (\neg v_{01} \lor \neg v_{31}) \land (\neg v_{01} \lor \neg v_{34}) \land (\neg v_{01} \lor \neg v_{41}) \land (\neg v_{01} \lor \neg v_{44}) \land (\neg v_{01} \lor \neg v_{52}) \land (\neg v_{01} \lor \neg v_{54}) \land (\neg v_{02} \lor \neg v_{03}) \land (\neg v_{01} \lor \neg v_{54}) \land (\neg v_{01} \lor (\neg v_{01} \lor \neg v_{54}) \land (\neg v_{01} \lor (\neg v_{01} \lor \neg v_{54}) \land (\neg v_{01} \lor (\neg v_{01} \lor \neg v_{54}) \land (\neg v_{01} \lor (\neg v$
$(\neg v_{02} \lor \neg v_{04}) \land (\neg v_{02} \lor \neg v_{05}) \land (\neg v_{02} \lor \neg v_{11}) \land (\neg v_{02} \lor \neg v_{12}) \land (\neg v_{02} \lor \neg v_{13}) \land (\neg v_{02} \lor \neg v_{14}) \land (\neg v_{02} \lor \neg v_{21}) \land (\neg v_{02} \lor \neg v_{22}) \land (\neg v_{02} \lor \neg v_{14}) \land (\neg v_{14} \lor (\neg v_{14} \lor \neg v_{14}) \land (\neg v_{14} \lor \neg v_$
$(\neg v_{02} \lor \neg v_{23}) \land (\neg v_{02} \lor \neg v_{24}) \land (\neg v_{02} \lor \neg v_{31}) \land (\neg v_{02} \lor \neg v_{32}) \land (\neg v_{02} \lor \neg v_{33}) \land (\neg v_{02} \lor \neg v_{41}) \land (\neg v_{02} \lor \neg v_{42}) \land (\neg v_{02} \lor \neg v_{43}) \land (\neg v_{02} \lor \neg v_{24}) \land (\neg v$
$(\neg v_{02} \lor \neg v_{53}) \land (\neg v_{03} \lor \neg v_{04}) \land (\neg v_{03} \lor \neg v_{05}) \land (\neg v_{03} \lor \neg v_{11}) \land (\neg v_{03} \lor \neg v_{14}) \land (\neg v_{03} \lor \neg v_{21}) \land (\neg v_{03} \lor \neg v_{24}) \land (\neg v_{03} \lor \neg v_{21}) \land (\neg v_{03} \lor (\neg v_{21}) \land (\neg v_{21} \lor (\neg v_{21}) \land (\neg v_{21}) \land (\neg v_{21} \lor (\neg v_{21}) \land (\neg v_{21}) \land (\neg v_{21} \lor (\neg v_{21}) \land (\neg v_{21}) \land (\neg v_{21} \lor (\neg v_{21}) \land (\neg v_{21}) \land (\neg v_{21} \lor (\neg v_{21}) \land (\neg v_{21} \lor (\neg v_{21}) \land (\neg v_{21} \lor (\neg v_{21}) \land (\neg v_{21}) \land (\neg v_{21} \lor (\neg v_{21}) \land (\neg$
$(\neg v_{03} \lor \neg v_{41}) \land (\neg v_{03} \lor \neg v_{53}) \land (\neg v_{04} \lor \neg v_{05}) \land (\neg v_{04} \lor \neg v_{11}) \land (\neg v_{04} \lor \neg v_{14}) \land (\neg v_{04} \lor \neg v_{21}) \land (\neg v_{04} \lor \neg v_{24}) \land (\neg v_{04} \lor \neg v_{21}) \land (\neg v_{04} \lor (\neg v_{21} \lor (\neg v$
$(\neg v_{04} \lor \neg v_{41}) \land (\neg v_{04} \lor \neg v_{53}) \land (\neg v_{10} \lor \neg v_{11}) \land (\neg v_{10} \lor \neg v_{12}) \land (\neg v_{10} \lor \neg v_{13}) \land (\neg v_{10} \lor \neg v_{14}) \land (\neg v_{10} \lor \neg v_{15}) \land (\neg v_{10} \lor \neg v_{22}) \land (\neg v_{10} \lor \neg v_{13}) \land (\neg v_{10} \lor \vee v_{13}) \land (\neg v$
$(\neg v_{10} \lor \neg v_{23}) \land (\neg v_{10} \lor \neg v_{32}) \land (\neg v_{10} \lor \neg v_{33}) \land (\neg v_{10} \lor \neg v_{41}) \land (\neg v_{10} \lor \neg v_{42}) \land (\neg v_{10} \lor \neg v_{43}) \land (\neg v_{10} \lor \neg v_{44}) \land (\neg v_{10} \lor \neg v_{51}) \land (\neg v_{10} \lor (\neg v_{10} \lor \neg v_{51}) \land (\neg v_{10} \lor \vee v_{51}) \land (\neg v_{10} \lor \vee v_{51}) \land (\neg v_{10} \lor \vee v_$
$(\neg v_{10} \lor \neg v_{52}) \land (\neg v_{10} \lor \neg v_{53}) \land (\neg v_{10} \lor \neg v_{54}) \land (\neg v_{10} \lor \neg v_{55}) \land (\neg v_{11} \lor \neg v_{12}) \land (\neg v_{11} \lor \neg v_{13}) \land (\neg v_{11} \lor \neg v_{14}) \land (\neg v_{11} \lor \neg v_{15}) \land (\neg v_{11} \lor \neg v_{12}) \land (\neg v_{11} \lor \neg v_{13}) \land (\neg v_{11} \lor \neg v_{15}) \land (\neg v_{11} \lor \vee v_{15}) \land (\neg v_{15} \lor \vee v_{15}) \land (\neg v_{15} \lor \vee v_{15}) \land (\neg v$
$(\neg v_{11} \lor \neg v_{21}) \land (\neg v_{11} \lor \neg v_{31}) \land (\neg v_{11} \lor \neg v_{34}) \land (\neg v_{11} \lor \neg v_{41}) \land (\neg v_{11} \lor \neg v_{53}) \land (\neg v_{12} \lor \neg v_{13}) \land (\neg v_{12} \lor \neg v_{14}) \land (\neg v_{12} \lor \neg v_{15}) \land (\neg v_{12} \lor \neg v_{13}) \land (\neg v_{13} \lor (\neg v_{13} \lor \neg v_{13}) \land (\neg v_{13} \lor (\neg v_{13} \lor \neg v_{13}) \land (\neg v_{13} \lor (\neg v$
$(\neg v_{12} \lor \neg v_{21}) \land (\neg v_{12} \lor \neg v_{22}) \land (\neg v_{12} \lor \neg v_{31}) \land (\neg v_{12} \lor \neg v_{32}) \land (\neg v_{12} \lor \neg v_{33}) \land (\neg v_{12} \lor \neg v_{34}) \land (\neg v_{12} \lor \neg v_{41}) \land (\neg v_{12} \lor \neg v_{42}) \land (\neg v_{12} \lor \neg v_{33}) \land (\neg v_{12} \lor \neg v_{13}) \land (\neg v_{13} \lor (\neg v_{13} \lor \neg v_{13}) \land (\neg v_{13} \lor (\neg v_{13} \lor \neg v_{13}) \land (\neg v_{13} \lor (\neg v_{13} \lor \neg v_{13}) \land (\neg v_{13} \lor (\neg v_{13} \lor (\neg v_{13} \lor \neg v_{13}) \land (\neg v_{13} \lor (\neg v_{13} \lor (\neg v_{13} \lor (\neg v$
$(\neg v_{12} \lor \neg v_{43}) \land (\neg v_{12} \lor \neg v_{44}) \land (\neg v_{12} \lor \neg v_{53}) \land (\neg v_{13} \lor \neg v_{14}) \land (\neg v_{13} \lor \neg v_{15}) \land (\neg v_{13} \lor \neg v_{21}) \land (\neg v_{13} \lor \neg v_{31}) \land (\neg v_{13} \lor \neg v_{34}) \land (\neg v_{13} \lor \neg v_{14}) \land (\neg v_{14} \lor \vee v_{14}) \land (\neg v_{14} \lor \vee v_{14}) \land (\neg v_{14} \lor \vee v_{14}) \land (\neg v$
$(\neg v_{13} \lor \neg v_{44}) \land (\neg v_{13} \lor \neg v_{53}) \land (\neg v_{14} \lor \neg v_{15}) \land (\neg v_{14} \lor \neg v_{22}) \land (\neg v_{14} \lor \neg v_{23}) \land (\neg v_{14} \lor \neg v_{31}) \land (\neg v_{14} \lor \neg v_{34}) \land (\neg v_{14} \lor \neg v_{41}) \land (\neg v_{14} \lor \neg v_{31}) \land (\neg v_{14} \lor \neg v_{14}) \land (\neg v_{14} \lor (\neg v_{14} \lor \neg v_{14}) \land (\neg v_{14} \lor (\neg v_{14} \lor (\neg v_{14} \lor \neg v_{14}) \land (\neg v_{14} \lor (\neg v$
$(\neg v_{14} \lor \neg v_{42}) \land (\neg v_{14} \lor \neg v_{43}) \land (\neg v_{14} \lor \neg v_{44}) \land (\neg v_{14} \lor \neg v_{53}) \land (\neg v_{20} \lor \neg v_{21}) \land (\neg v_{20} \lor \neg v_{22}) \land (\neg v_{20} \lor \neg v_{23}) \land (\neg v_{20} \lor \neg v_{24}) \land (\neg v_{20} \lor \neg v_{21}) \land (\neg v_{20} \lor \neg v_{22}) \land (\neg v_{21} \lor (\neg v_{21} \lor \neg v_{21}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{21} \lor (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{21} \lor (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{21} \lor (\neg v_{21} \lor (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{21} \lor (\neg v_{21} \lor (\neg v_{21} \lor (\neg v$
$(\neg v_{20} \lor \neg v_{25}) \land (\neg v_{20} \lor \neg v_{32}) \land (\neg v_{20} \lor \neg v_{33}) \land (\neg v_{20} \lor \neg v_{41}) \land (\neg v_{20} \lor \neg v_{42}) \land (\neg v_{20} \lor \neg v_{43}) \land (\neg v_{20} \lor \neg v_{53}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{22} \lor \neg v_{33}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{22} \lor \neg v_{33}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{22} \lor \neg v_{33}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{22} \lor \neg v_{33}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{22} \lor \neg v_{33}) \land (\neg v_{21} \lor \neg v_{22}) \land (\neg v_{22} \lor \neg v_{33}) \land (\neg v_{23} \lor (\neg v_{23} \lor \neg v_{33}) \land (\neg v_{23} \lor (\neg v_{23} \lor \neg v_{33}) \land (\neg v_{23} \lor (\neg v_{23} \lor \neg v_{33}) \land (\neg v_{23} \lor (\neg v_{23} \lor (\neg v_{23} \lor \neg v_{33}) \land (\neg v_{23} \lor (\neg v$
$(\neg v_{21} \lor \neg v_{23}) \land (\neg v_{21} \lor \neg v_{24}) \land (\neg v_{21} \lor \neg v_{25}) \land (\neg v_{21} \lor \neg v_{31}) \land (\neg v_{21} \lor \neg v_{41}) \land (\neg v_{21} \lor \neg v_{44}) \land (\neg v_{21} \lor \neg v_{53}) \land (\neg v_{22} \lor \neg v_{23}) \land (\neg v_{22} \lor \neg v_{23}) \land (\neg v_{21} \lor \neg v_{24}) \land (\neg v_{21} \lor \neg v_{24}) \land (\neg v_{21} \lor \neg v_{24}) \land (\neg v_{21} \lor \neg v_{23}) \land (\neg v_{22} \lor \neg v_{23}) \land (\neg v_{22} \lor \neg v_{23}) \land (\neg v_{21} \lor \neg v_{24}) \land (\neg v_{21} \lor \neg v_{24}) \land (\neg v_{22} \lor \neg v_{23}) \land (\neg v_{23} \lor (\neg v_{23} \lor \neg v_{23}) \land (\neg v_{23} \lor (\neg v$
$(\neg v_{22} \lor \neg v_{24}) \land (\neg v_{22} \lor \neg v_{25}) \land (\neg v_{22} \lor \neg v_{31}) \land (\neg v_{22} \lor \neg v_{32}) \land (\neg v_{22} \lor \neg v_{41}) \land (\neg v_{22} \lor \neg v_{42}) \land (\neg v_{22} \lor \neg v_{43}) \land (\neg v_{22} \lor \neg v_{53}) \land (\neg v_{23} \lor \neg v_{53}) \land (\neg v_{23} \lor \neg v_{53}) \land (\neg v_{23} \lor (\neg v_{23} \lor \neg v_$
$(\neg v_{23} \lor \neg v_{24}) \land (\neg v_{23} \lor \neg v_{25}) \land (\neg v_{23} \lor \neg v_{31}) \land (\neg v_{23} \lor \neg v_{41}) \land (\neg v_{23} \lor \neg v_{43}) \land (\neg v_{24} \lor \neg v_{25}) \land (\neg v_{24} \lor \neg v_{32}) \land (\neg v_{24} \lor \neg v_{33}) \land (\neg v_{24} \lor (\neg v_{33} \lor (\neg v_{33}) \land (\neg v_{33} \lor (\neg v_{33} \lor (\neg v_{33}) \land (\neg v_{33} \lor (\neg v_{33} \lor (\neg v_{33}) \land (\neg v_{33} \lor ($
$(\neg v_{24} \lor \neg v_{41}) \land (\neg v_{24} \lor \neg v_{44}) \land (\neg v_{24} \lor \neg v_{53}) \land (\neg v_{30} \lor \neg v_{31}) \land (\neg v_{30} \lor \neg v_{32}) \land (\neg v_{30} \lor \neg v_{33}) \land (\neg v_{30} \lor \neg v_{34}) \land (\neg v_{30} \lor \neg v_{35}) \land (\neg v_{30} \lor \neg v_{31}) \land (\neg v_{30} \lor (\neg v_{31}) \land (\neg v_{30} \lor \neg v_{31}) \land (\neg v_{30} \lor (\neg v_{31}) \land (\neg v_{30} \lor (\neg v_{31}) \land (\neg v_{31}) \land (\neg v_{31} \lor (\neg v_{31}) \land (\neg v_{31} \lor (\neg v_{31}) \land (\neg v_{31}) \land (\neg v_{31} \lor ($
$(\neg v_{31} \lor \neg v_{32}) \land (\neg v_{31} \lor \neg v_{33}) \land (\neg v_{31} \lor \neg v_{34}) \land (\neg v_{31} \lor \neg v_{35}) \land (\neg v_{31} \lor \neg v_{41}) \land (\neg v_{31} \lor \neg v_{44}) \land (\neg v_{31} \lor \neg v_{52}) \land (\neg v_{31} \lor \neg v_{53}) \land (\neg v_{31} \lor (\neg v_{31} \lor \neg v_{53}) \land (\neg v_{53} \lor (\neg v$
$(\neg v_{32} \lor \neg v_{33}) \land (\neg v_{32} \lor \neg v_{34}) \land (\neg v_{32} \lor \neg v_{35}) \land (\neg v_{32} \lor \neg v_{51}) \land (\neg v_{32} \lor \neg v_{54}) \land (\neg v_{33} \lor \neg v_{34}) \land (\neg v_{33} \lor \neg v_{35}) \land (\neg v_{33} \lor \neg v_{40}) \land (\neg v_{33} \lor \neg v_{35}) \land (\neg v_{35} \lor (\neg v_{35} \lor \neg v_{35}) \land (\neg v_{35} \lor (\neg v_{35} \lor \neg v_{35}) \land (\neg v_{35} \lor (\neg v_{35} \lor \neg v_{35}) \land (\neg v_{35} \lor (\neg v_{35} \lor \vee v_{35}) \land (\neg v_{35} \lor (\neg v_{35} \lor \vee v_{35}) \land (\neg v_{35} \lor (\neg v$
$(\neg v_{33} \lor \neg v_{45}) \land (\neg v_{33} \lor \neg v_{51}) \land (\neg v_{33} \lor \neg v_{54}) \land (\neg v_{34} \lor \neg v_{35}) \land (\neg v_{34} \lor \neg v_{41}) \land (\neg v_{34} \lor \neg v_{42}) \land (\neg v_{34} \lor \neg v_{43}) \land (\neg v_{34} \lor \neg v_{44}) \land (\neg v_{34} \lor \neg v_{45}) \land (\neg v_{45} \lor (\neg v_{45} \lor v_{45}) \land (\neg$
$(\neg v_{34} \lor \neg v_{52}) \land (\neg v_{34} \lor \neg v_{53}) \land (\neg v_{40} \lor \neg v_{41}) \land (\neg v_{40} \lor \neg v_{42}) \land (\neg v_{40} \lor \neg v_{43}) \land (\neg v_{40} \lor \neg v_{44}) \land (\neg v_{40} \lor \neg v_{45}) \land (\neg v_{41} \lor \neg v_{42}) \land (\neg v_{40} \lor \neg v_{43}) \land (\neg v_{40} \lor \vee v_{43}) \land (\neg v$
$(\neg v_{41} \lor \neg v_{43}) \land (\neg v_{41} \lor \neg v_{44}) \land (\neg v_{41} \lor \neg v_{52}) \land (\neg v_{41} \lor \neg v_{52}) \land (\neg v_{41} \lor \neg v_{53}) \land (\neg v_{42} \lor \neg v_{43}) \land (\neg v_{42} \lor \neg v_{44}) \land (\neg v_{42} \lor \neg v_{45}) \land (\neg v_{42} \lor \neg v_{43}) \land (\neg v_{43} \lor (\neg v_{43} \lor (\neg v_{43}) \land (\neg v_{43} \lor (\neg v_{43} \lor (\neg v_{43}) \land (\neg v_$
$(\neg v_{42} \lor \neg v_{51}) \land (\neg v_{42} \lor \neg v_{54}) \land (\neg v_{43} \lor \neg v_{44}) \land (\neg v_{43} \lor \neg v_{45}) \land (\neg v_{43} \lor \neg v_{51}) \land (\neg v_{43} \lor \neg v_{54}) \land (\neg v_{44} \lor \neg v_{45}) \land (\neg v_{44} \lor \neg v_{52}) \land (\neg v_{43} \lor \neg v_{51}) \land (\neg v_{43} \lor \neg v_{51}) \land (\neg v_{44} \lor \neg v_{52}) \land (\neg v_{44} \lor \neg v_{51}) \land (\neg v_{54} \lor (\neg v_{51} \lor (\neg v$
$(\neg v_{44} \lor \neg v_{53}) \land (\neg v_{50} \lor \neg v_{51}) \land (\neg v_{50} \lor \neg v_{52}) \land (\neg v_{50} \lor \neg v_{53}) \land (\neg v_{50} \lor \neg v_{54}) \land (\neg v_{50} \lor \neg v_{55}) \land (\neg v_{51} \lor \neg v_{52}) \land (\neg v_{51} \lor \neg v_{53}) \land (\neg v_{51} \lor \vee v_{53}) \land (\neg v_{53} \lor (\neg v_{53} \lor \vee v_{53}) \land (\neg v_{53} \lor (\neg v_{53} \lor \vee v_{53}) \land (\neg v_{53} \lor (\neg v_{53} \lor \vee v_{53}) \land (\neg v_{53} \lor (\neg v_{53} \lor (\neg v_{53} \lor \vee v_{53}) \land (\neg v_{53} \lor (\neg v$
$(\neg v_{51} \lor \neg v_{54}) \land (\neg v_{51} \lor \neg v_{55}) \land (\neg v_{52} \lor \neg v_{53}) \land (\neg v_{52} \lor \neg v_{54}) \land (\neg v_{52} \lor \neg v_{55}) \land (\neg v_{53} \lor \neg v_{54}) \land (\neg v_{53} \lor \neg v_{55}) \land (\neg v_{54} \lor \neg v_{55})$

figure 7.1: CNF formula without at-least-1 clauses